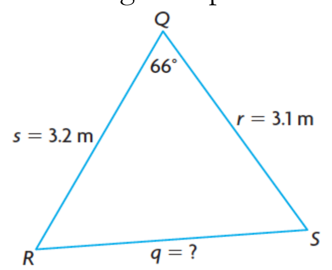
# Chapter 3- Acute Triangle Trigonometry - Lesson 4- 3.3 Cosine Law

**GOAL:** To use the cosine law to solve for missing sides and missing angles in triangles where the Sine Law will not work.

The sine law cannot always help you determine the sides or angles in a triangle. Consider the following example:




$$\frac{\sin Q}{q} = \frac{\sin S}{s} = \frac{\sin R}{r}$$

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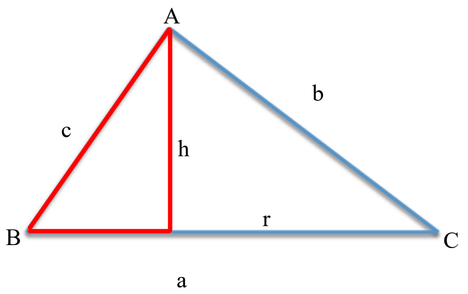
**Proof of Cosine Law:** We can use the <u>Pythagorean Theorem</u> theorem to prove the Cosine Law




$\sin C = \frac{h}{b}$ $\cos C = \frac{r}{b}$

$h = b \sin C$ $r = b \cos C$

**<u>Pythagorean Theorem (red triangle)</u>**

$c^2 = h^2 + (a-r)^2$

$c^2 = (b\sin C)^2 + (a - b\cos C)^2$ **Substitute "h" and "r" for above numbers.

$(a - b\cos C)(a - b\cos C)$

$= a^2 - ab\cos C - ab\cos C + b^2(\cos C)^2$

$= a^2 - 2ab\cos C + b^2(\cos C)^2$

$c^2 = b^2(\sin C)^2 + a^2 - 2ab\cos C + (b^2\cos C)^2$

$c^2 = b^2(\sin C)^2 + b^2(\cos C)^2 + a^2 - 2ab\cos C$

$c^2 = b^2(\sin C^2 + \cos C^2) + a^2 - 2ab\cos C$ **factor out $b^2$

$c^2 = b^2(1) + a^2 - 2ab\cos C$

**Try it with an angle! → $\sin C^2 + \cos C^2 = 1$

$$\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$$

Cosine Law

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## <u>EXTREMELY IMPORTANT</u>

$$\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$$

*It is extremely important to recognize that the first letter in this formula does not need to be "c" but can be any letter in your triangle.

- The beginning letter and letter that follows Cos <u>MUST</u> be the same
- The first two letters following the = sign must be the other 2 letters not used

<u>This means your formula for a portential triangle can look like any of the following:</u>

**<u>Triangle ABC</u>**

1) $c^2 = a^2 + b^2 - 2ab\cos C$

**<u>OR</u>**

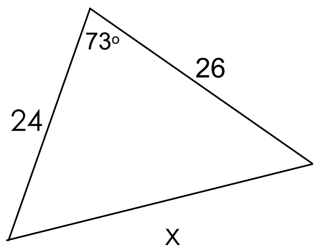
**<u>Triangle RST</u>**

1)

2)

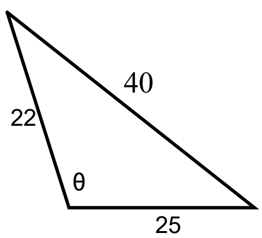
3)

Using Cosine Law to find the length of sides:
When to use: Look for a "Sandwich!" (2 lines sandwiched between an angle)




Using Cosine Law to find angles: When all three sides are given

We can rearrange the cosine law before substituting. Remember we are finding an angle, so we must use the INVERSE COS function.

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TRY: In $\Delta DJG$, DG is 8, JG is 11 and DJ is 15. Solve for D.

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