Chapter 5 – Systems of Linear Inequalities
Section 5.1 Graphing Linear Inequalities in Two Variables

RF1: Model and solve problems that involve systems of linear inequalities in two variables.

Frank and Joe sell lemonade. They buy lemons for \$.50 each and sugar costs them \$1 for 1kg. They have \$20 to spend on supplies.

a. Give two combinations of lemons and sugar that would total \$20

20 lemons – 20 x \$0.50 = \$10

10 Kg of sugar – 10 x \$1 = \$10

b. Represent this situation with an equation

$0.5L + 1S = 20$

L = # of lemons

S = kg of sugar

<> as an inequality → $0.5L + 1S \leq 20$

c. What is the domain (lemons) and range (sugar) for each?

Lemons – $\{L \mid \boxed{0 \leq L \leq 40},\ L \in I\}$

Sugar – $\{S \mid 0 \leq S \leq 20,\ S \in I\}$

d) Graph the relationship

< 20

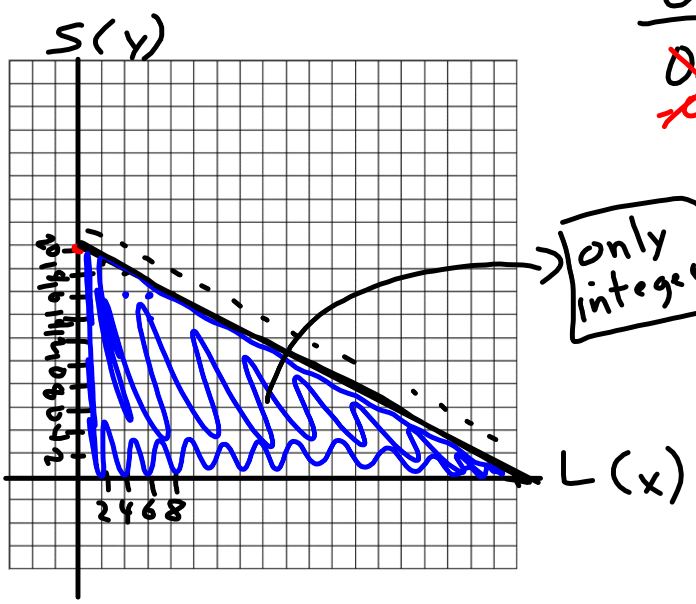
$0.5L + 1S \leq 20$

$\cancel{0.5x} + 1y \leq 20 - 0.5x$

$\cancel{-0.5x}$

$y \leq -0.5x + 20$

$\boxed{y \leq -\frac{1}{2}x + 20}$



only integers

e) What REGION of the graph represents different possibilities, especially if they spend less than \$20

– everything below the line.

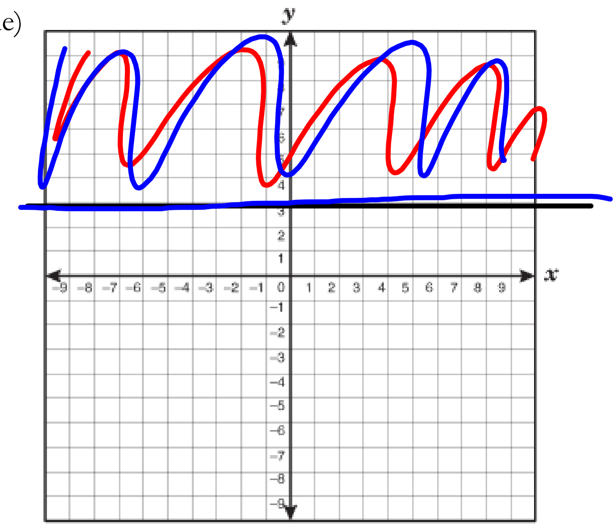
f. Is the solution set represented by the region above the line, below the line, or on the line itself?

" "

g. Is the line solid or dashed?

Solid.

Look at the graph of $y = 3$ (straight horizontal line)



The line divides the plane into two halves:

· $y < 3$ is the region below the line.

· $y > 3$ is the region above the line.

· $y = 3$ is the boundary line.

A solid boundary line is used to represent $\leq$ or $\geq$.

A dashed boundary line is used to represent $<$ or $>$

~~A ____________ boundary line is used to represent $\leq$ or $\geq$ when the values are also discrete (ie: only integers)~~

To graph an inequality:

1. Graph the boundary line (as if it were a normal equation).

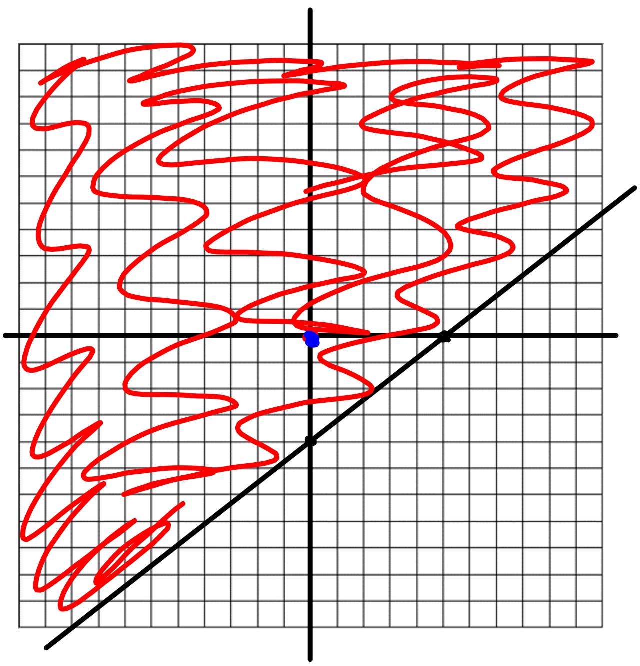
2. Pick a point (x,y) not on the line and substitute it into the inequality.

3. If the inequality is satisfied (true), shade the region containing the point. If not, shade the other region.

Example 1: Graph $4x - 5y \leq 20$

$$4x - 5y = 20$$
$$\cancel{-4x} \qquad -4x$$
$$\frac{-5y}{-5} = \frac{-4x + 20}{-5}$$

$$y = \frac{4}{5}x - 4$$



Test (0,0)

$4(0) - 5(0) \leq 20$

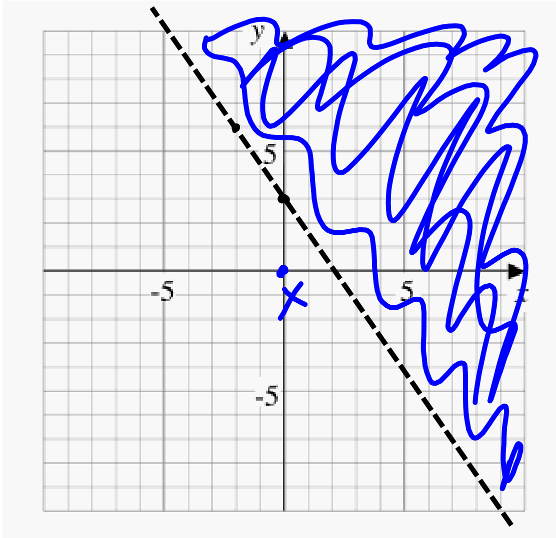
$\boxed{0 \leq 20}$ ✓

Example 2: Graph the following $y > -\frac{3}{2}x + 3$

Test (0,0)

$(0) > -\cancel{\frac{3}{2}}\cancel{(0)} + 3$

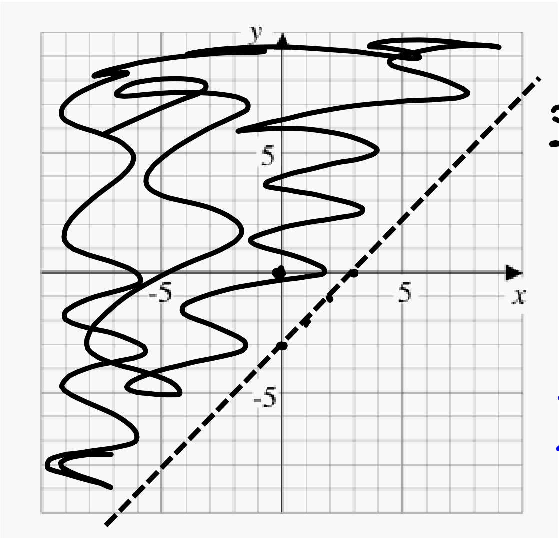
$\boxed{0 > 3}$ X



Example 3: Graph the solution set for each linear inequality on a Cartesian plane:

a) $\{(x, y) \mid \boxed{y > x - 3}, x \in R, y \in R\}$

b) $\{(x, y) \mid \boxed{2x + 3y \geq 6}, x \in I, y \in I\}$



Test (0,0)

$\cancel{2(0)} + \cancel{3(0)} \geq 6$

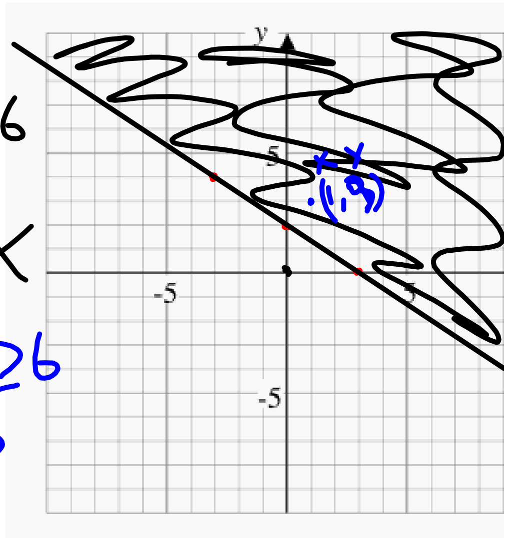
$0 \quad 0$

$\boxed{0 \geq 6}$ X

$2(1) + 3(3) \geq 6$

$2 + 9 \geq 6$

$\boxed{11 \geq 6}$



$\cancel{2x} + 3y \geq 6 - 2x$

$-\cancel{2x}$

$\frac{\cancel{3}y}{\cancel{3}} \geq \frac{-2x}{3} + \frac{6}{3}$

$\boxed{y \geq \frac{-2}{3} + 2}$

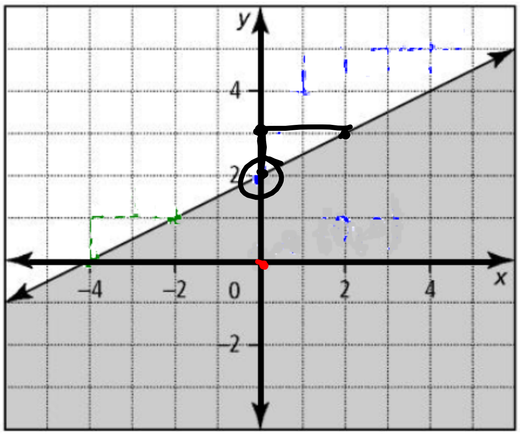
What is the difference??

Test (0,0)

$(0) > (0) - 3$

$\boxed{0 > -3}$ ✓

Example 4: Write an inequality to represent each graph (Hint: y = mx + b...Find slope 1st):

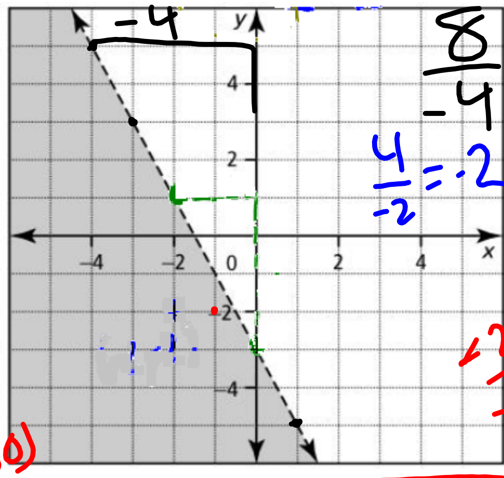


$y \le \frac{1}{2}x + 2$

Test (0,0)

$0 \ge \frac{1}{2}(0) + 2$

$0 \ngeq 2$ $\le$



$\frac{8}{-4} = -2$

$\frac{4}{-2} = -2$

$y = mx + b$

$\frac{-2}{1} = -2$

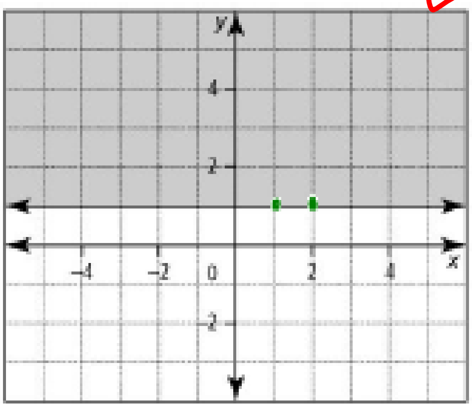
$\boxed{y < -2x - 3}$

Test (-1, -2)

$(-2) > -2(-1) - 3$

$-2 > +2 - 3$

$\boxed{-2 > -1}$



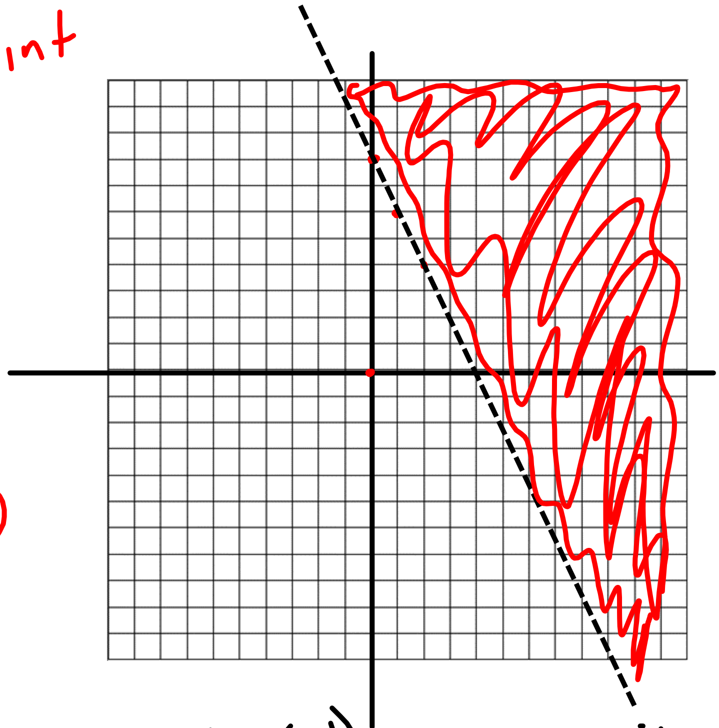
page 221 # 1b, 2 (use 2x+3y>6), 4, 5af,

$y = mx + b$

5a) $y > -2x + 8$

y int

$m = \frac{-2}{1}$



Test (0,0)

$(0) > \cancel{-2(0)} + 8$

$\boxed{0 > 8}$

(y int x=(0))

$4(0) + \frac{3y}{3} = \frac{-12}{3}$

$\boxed{y = -4}$

x int (y=0)

$\frac{4x}{4} + \cancel{3(0)} = \frac{-12}{4}$

$\boxed{x = -3}$

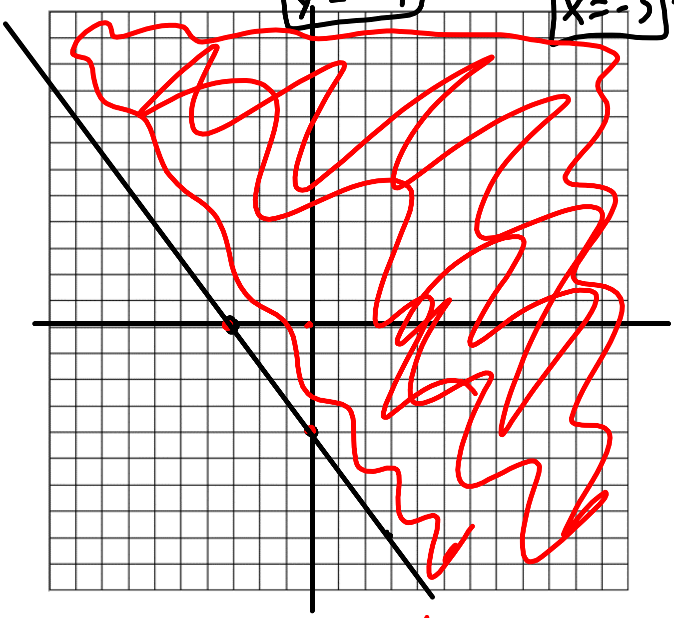
$y = mx + b$

5f) $\boxed{4x + 3y \geq -12}$

$-4x \qquad -4x$

$\frac{3y}{3} \geq \frac{-4x}{3} - \frac{12}{3}$

$\boxed{y \geq -\frac{4}{3}x - 4}$



Test (0,0)

$4(0) + 3(0) \geq -12$

$\boxed{0 \geq -12}$ ✓

$0 \geq -\frac{4}{3}(0) - 4$

$\boxed{0 \geq -4}$ ✓

page 221 # 1b, 2 (use 2x+3y>6), 4, 5af,

1. Graph the solution set for each linear inequality.

a) $y < x + 4$ b) $-y < -6x + 3$

2. Consider the graph of this inequality.

$$2x + 3y > 6$$

Make each of the following decisions, and provide your reasoning.

a) whether the boundary should be dashed, stippled, or solid

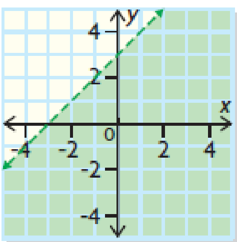
b) whether the half plane above or below the boundary should be shaded

c) whether each point is in its solution region:

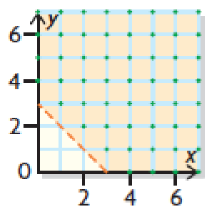
i) (1, 1) ii) (1, 0) iii) (1, 2)

4. Match each graph with its linear inequality, and justify your match.

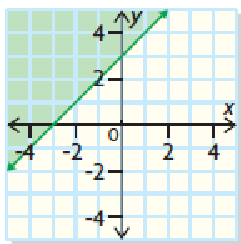
a)



b)



c)



i) $\{(x, y) \mid x - 3 > -y, x \in W, y \in W\}$

ii) $\{(x, y) \mid x - y > -3, x \in R, y \in R\}$

iii) $\{(x, y) \mid y - 3 \geq x, x \in R, y \in R\}$

5. Graph the solution set for each linear inequality.

a) $y > -2x + 8$

b) $-3y \leq 9x + 12$

c) $y < 6$

d) $-4x - 8 > 4$

e) $10x - 12 < -y$

f) $4x + 3y \geq -12$

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RF1: Model and solve problems that involve systems of linear inequalities in two variables.

Word Problem Examples

Example 1: Ben is buying snacks for his friends. He has $10.00. The choices are apples for $0.80 and muffins for $1.25.

a) Define your variables. Write an inequality to model this situation.

x - apples
y - muffins

$\boxed{0.8x + 1.25y \le 10}$

b) State the restrictions on the variables (ie: Domain and Range).

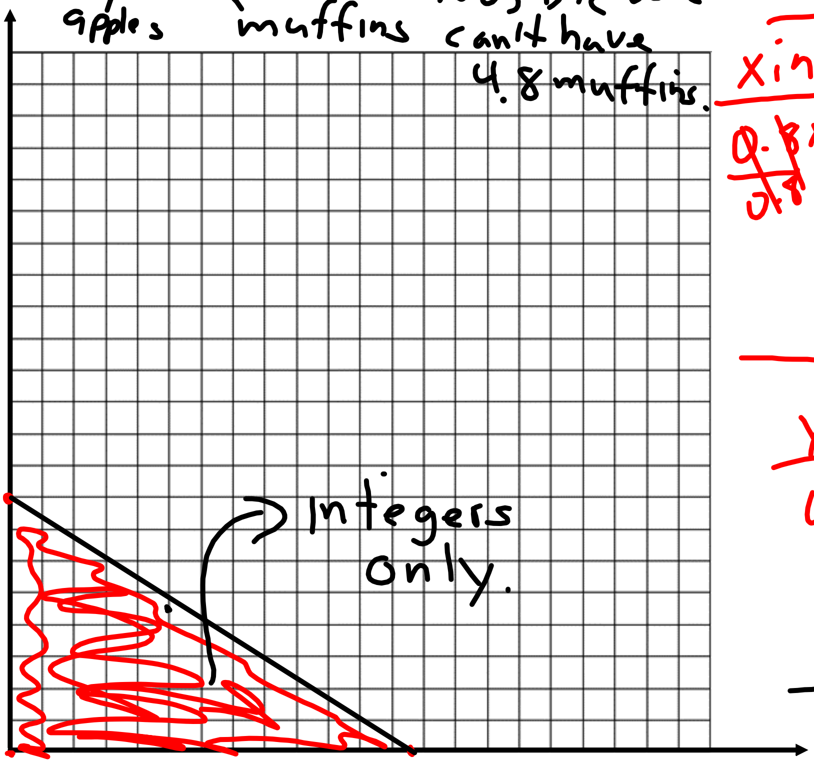
$\{x \mid 0 \le x \le 12,\ x \in I\}$
$\{y \mid 0 \le y \le 8,\ x \in I\}$

c) Graph the inequality.

d) Is (5, 4.8) a possible solution? Why or why not?

apples   muffins

No, b/c we can't have 4.8 muffins.



$0.8x + 1.25y = 10$

xint (y = 0)

$\frac{0.8x}{0.8} + \cancel{1.25(0)} = \frac{10}{0.8}$

$x = 12.5$

yint (x = 0)

$\cancel{0.8(0)} + \frac{1.25y}{1.25} = \frac{10}{1.25}$

$\boxed{y = 8}$

Test (0,0)

$0 + 0 \le 10$

$\boxed{0 \le 10}$ ✓

Example 2: A sports store has a net revenue of $100 on every pair of downhill skis sold and $120
on every snowboard sold. The manager's goal is to have a net revenue of more than $600 a day from the sales of these two items. What combinations of ski and snowboard sales will accomplish this daily sales goal?

a) Define your variables. Write an inequality to model this situation.

x - skis
y - snowboards

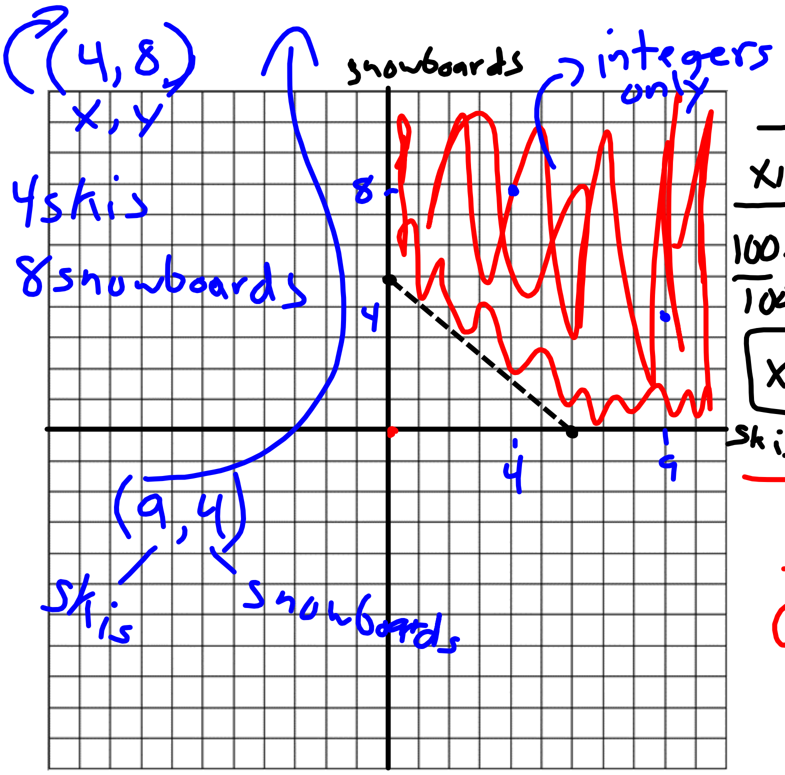
$$\boxed{100x + 120y > 600}$$

b) State the restrictions on the variables.

$\{x \mid 0 \leq x,\ x \in I\}$
$\{y \mid 0 \leq y,\ y \in I\}$

c) Graph the inequality.

d) List two possible combinations that are true and explain what they mean.



$100x + 120y = 600$

x int (y=0)

$100x + \cancel{120(0)} = 600$

$\frac{100x}{100} = \frac{600}{100}$

$\boxed{x = 6}$

y int (x=0)

$\cancel{100(0)} + 120y = 600$

$\frac{120y}{120} = \frac{600}{120}$

$\boxed{y = 5}$

Test (0,0)

$0 + 0 > 600$

$\boxed{0 > 600}$ ✗

Class/Homework: page 222 # 9, 10, 12

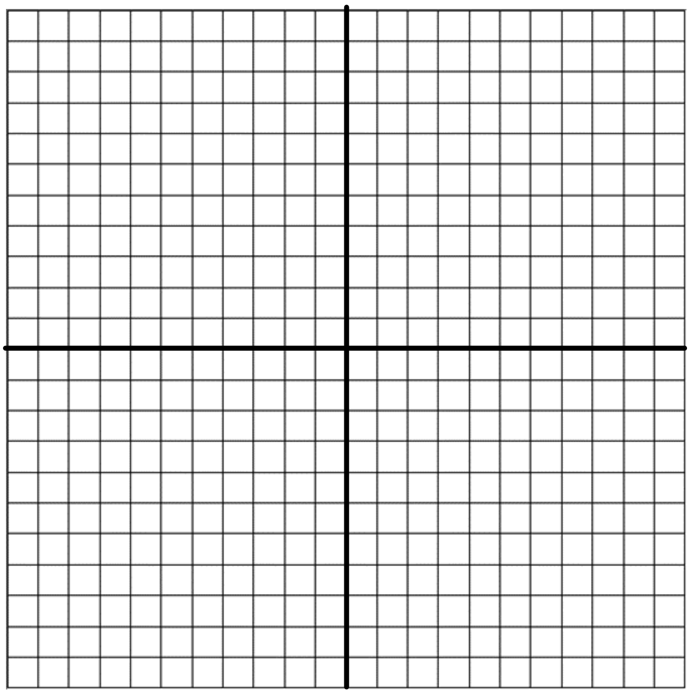
9. For every teddy bear that is sold at a fundraising banquet, $10 goes to charity. For every ticket that is sold, $32 goes to charity. The organizers' goal is to raise at least $5000. The organizers need to know how many teddy bears and tickets must be sold to meet their goal.

a) Define the variables and write a linear inequality to represent the situation.

b) What are the restrictions on the variables? How do you know?

c) Graph the linear inequality to help you determine whether each of the following points is in the solution set. The first coordinate is the number of teddy bears and the second is the number of tickets.

i) (400, 20) **ii)** (205, 98) **iii)** (156, 105)



10. On Earth Day, a nursery sold more than \$1500 worth of maple and birch trees. The maple trees were sold for \$75, and the birch trees were sold for \$50.

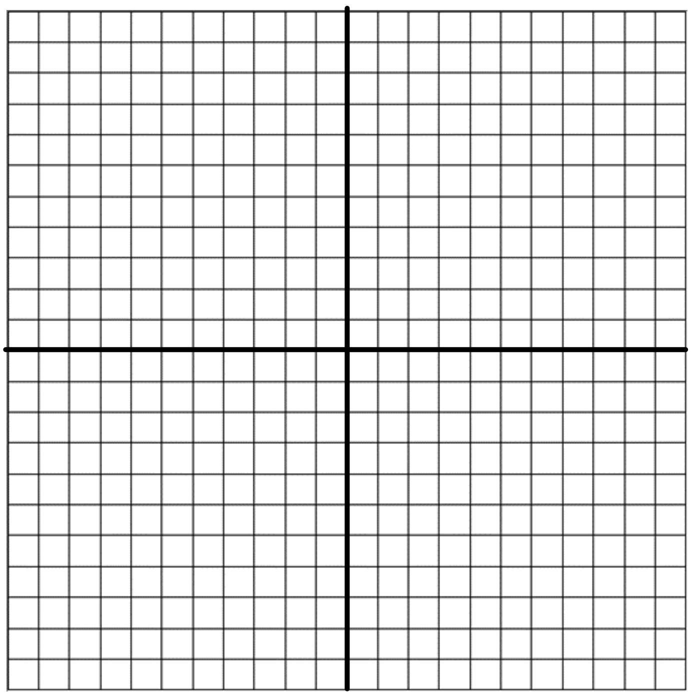
a) Define the variables and write a linear inequality to represent the possible combinations of trees sold. Are there any restrictions on the variables? Explain.

b) Graph the linear inequality.

c) Use your graph to determine:

i) if the nursery could have sold 13 of each type of tree

ii) if 14 of one type and 9 of the other type could have been sold



12. A banquet room is set up to seat, at most, 660 people. Each rectangular table seats 12 people, and each circular table seats 8 people.

a) Define the variables and write a linear inequality to represent the number of each type of table needed. Then graph your inequality.

b) The organizers of the banquet would like to have as close to the same number of rectangular tables and circular tables as possible. What combination of tables could they use? Explain your choice.