5.4 Optimization Problems I: Creating the Model ----- GOAL: *Create models to represent optimization problems*

Example 1: A toy company manufactures two types of toy vehicles: racing cars and sport-utility vehicles.

- Supply of materials is limited so no more than 40 racing cars and 60 sport utility vehicles can be made daily.
- However, the company can make 70 or more vehicles, in total, each day.
- It costs \$8 to make a racing car and \$12 to make a sport-utility vehicle.

The company wants to know what combinations of racing cars and SUV's will result in the minimum and maximum costs, and what those costs will be.

How can this situation be modelled?

A. Define the Variables

x =

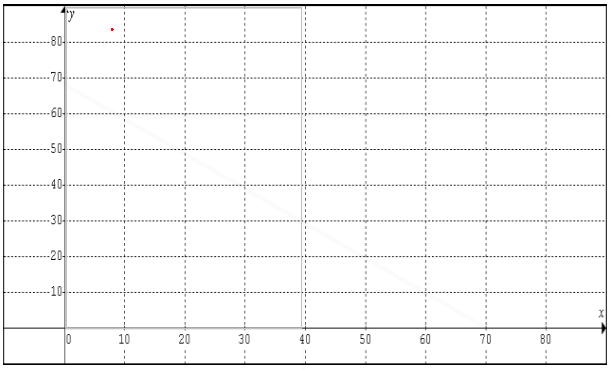
y =

B. Write a system of linear inequalities to represent these conditions:

- The total number of racing cars that can be made:
- The total number of sport-utility vehicles that can be made:
- The total number of vehicles that can be made:

C. Graph the system.

3) Test (0,0)



E. What quantity in this situation needs to be minimized and maximized? ____________

Write an equation to represent how the two variables relate to this quantity.

F. Each combination below is a possible solution to the system of linear inequalities graphed above:

(i) 40 racing cars and 30 sport-utility vehicles

(ii) 10 racing cars and 60 sport-utility vehicles

(iii) 40 racing cars and 60 sport-utility vehicles

Where do these values occur on your graph? ______________________________

Use your equation from part E to calculate the manufacturing cost for each situation.

i) $C =$

ii) $C =$

iii) $C =$

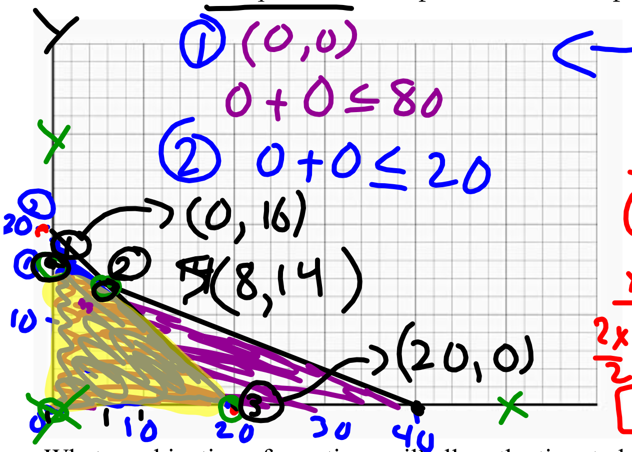
Where do minimum and maximum costs occur according to your equation above?

Maximum- ______________________________

Minimum - ______________________________

x - MC
y - OE

Example 2: A test is made up of multiple-choice and open-ended questions. It takes up to 2 minutes to do a multiple-choice question and up to 5 minutes for an open ended question. The total time for the test is 80 minutes and you may answer no more than 20 questions. Graph the solution space and label each corner point.



① $2x + 5y \le 80$ *

② $\boxed{x + y \le 20}$

① $2x + 5y = 80$

x int: $2x + 5(0) = 80$, $\boxed{x = 40}$

y int: $2(0) + 5y = 80$, $\boxed{y = 16}$

② $x = 20$, $y = 20$

What combination of questions will allow the time to be maximized to fill up the majority of the class?

- To find this we need to create what is called an "Objective Function" to find our possibilities.

What value are we trying to maximize here? Time

Write an equation that could be used to find the total time? >>>> $\boxed{\text{Time} = 2x + 5y}$ *(this is known as an "Objective Function")*

Where will our maximum occur on the graph above? Intersections

How many points do we need to test then? 3 b/c (0,0) will not maximize.

Test Them $T = 2x + 5y$

① (0, 16) *

② (8, 14) → $2(8) + 5(14) = 16 + 70 = \boxed{86}$ min

③ (20, 0)

Which one will result in the maximum time? __________ *this is your "Optimized solution"

Fred is planning an exercise program where he wants to run and swim every week. He doesn't want to spend more than 12 hours a week exercising and he wants to burn at least 1600 calories a week. Running burns 200 calories an hour and swimming burns 400 calories an hour. Running costs $1 an hour while swimming costs $2 an hour. How many hours should he spend at each sport to keep his costs at a minimum?

a) Define Variables

x – running hours

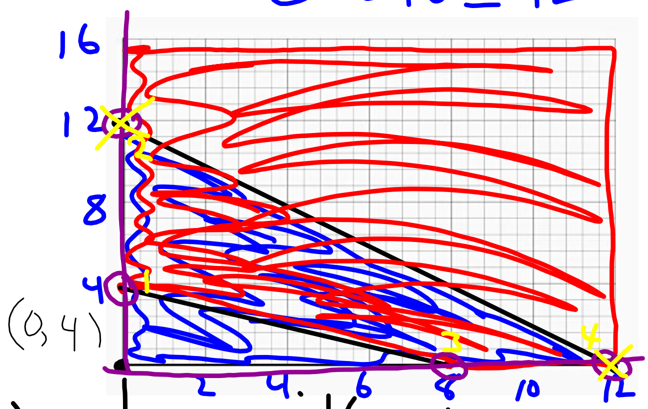
y – swimming hours

b) Create Inequalities

① $\boxed{x + y \le 12}$

② $200x + 400y \ge 1600$

① $0 + 0 \le 12$



c) Graph Inequalities

① $x = 12$
$y = 12$

② xint (y=0)

$$\frac{\cancel{200}x}{\cancel{200}} = \frac{1600}{200}$$

$$\boxed{x = 8}$$

yint (x=0)

$$200(0) + 400y = 1600$$

$$\cancel{0} + \frac{\cancel{400}y}{\cancel{400}} = \frac{1600}{400}$$

$$\boxed{y = 4}$$

d) Create "Objective Function" *(what do we want to maximize or minimize?)*

Cost = 1x + 2y

e) How many points do we have to test in our minimum cost equation in this example? _______
Test each one to find your minimum.

* ① $C = 1x + 2y = 1(0) + 2(4) = 0 + 8 = \$8$ ① (0, 4)

② $C = 1x + 2y = 1(8) + 2(0) = 8 + 0 = \$8$ ② (8, 0)

To minimize cost he should swim for 4 hour (0 hr running) or 8 hrs running and 0 swimming.

Assignment pg. 248-9 Questions: 2, 3, 4, and 5